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SIMULTANEOUS HEAT AND MASS TRANSFER BETWEEN GAS AND LIQUID PHASES

II.—ANALYSIS OF STEADY STATE TRANSFER

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Abstract—The method of analysis by the double film model is extended to the general case of the air–aqueous solution system, and the relations among the transfer coefficients of gas and liquid film are derived. Furthermore, the application of the dehumidification operation to the design of the equipment is discussed.

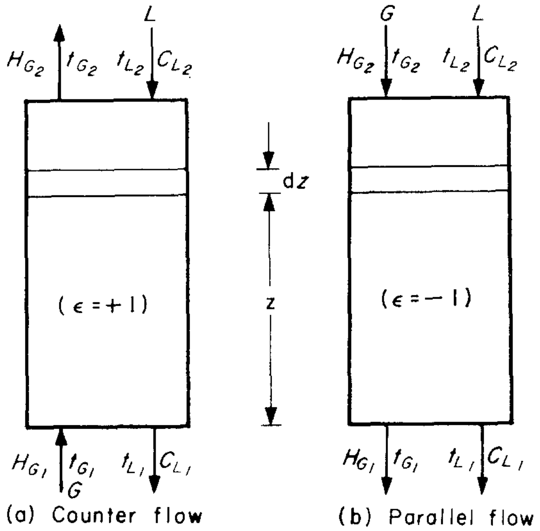
NOMENCLATURE

a,	effective contact area between gas and liquid [m^2/m^3];
C_H,	humid heat of gas [kcal/kg degC];
C_i,	concentration at the interface between gas and liquid [kg-water/kg-solvent];
C_s,	specific heat capacity of liquid [kcal/kg degC];
G,	gas flow rate [kg-dry air/h];
H,	absolute humidity of gas [kg-water/kg-dry air];
$H_{Gm}, H_{Gh}, H_{Lm}, H_{Lh}$,	height of a transfer unit [m];
h_G,	gas film coefficient for heat transfer [kcal/h m^2 degC];
h_L,	liquid film coefficient for heat transfer [kcal/h m^2 degC];
i_G,	enthalpy of gas [kcal/kg];
i_i,	enthalpy at the interface between gas and liquid [kcal/kg];
i_L,	enthalpy of liquid [kcal/kg];
k_G,	gas film coefficient for mass transfer [kg/h m^2 ΔH];
k_L,	liquid film coefficient for mass transfer [kg/h m^2 ΔC];
K_{OG},	overall mass-transfer coefficient taking the enthalpy difference as driving force [kcal/h m^2 Δi];
L,	liquid flow rate [kg-solvent/h];
m,	constant in equation (31);
m',	constant in equation (30);
n,	constant in equation (31);
n',	constant in equation (30);
$N_{Gh}, N_{Gm}, N_{Lh}, N_{Lm}$,	number of transfer unit;
N,	overall number of transfer unit;
Q,	total transfer rate of enthalpy [kcal/h];
Q_s,	transfer rate of sensible heat [kcal/h];
Q_L,	transfer rate of latent heat [kcal/h];
γ,	latent heat [kcal/kg];
S,	cross section area of equipment [m^2];

t_G,	temperature of gas [°C];
t_i,	temperature at the interface [°C];
t_L,	bulk temperature of liquid [°C];
W,	mass-transfer rate [kg/h];
Z,	height of equipment [m];
α,	ratio of total transfer rate to sensible heat-transfer rate;
ϵ,	+ counter flow;
ϵ,	− parallel flow.

INTRODUCTION

IN THE previous paper (1), the equations of unsteady state heat and mass transfer were simultaneously solved and the mechanism of simultaneous transfer phenomena was discussed. To design the equipment used in humidification or cooling water operation, the analysis by the double film model has been conveniently applied. Simultaneous heat and mass transfer of the air–water system have been studied by several workers. For the air–water system, Mickely [1] and the other investigators proposed the method using the chart of enthalpy-temperature and the relations among the film coefficients. For the gas–liquid system for which the relations of Lewis are not valid, Bras [2], Mizusina *et al.* [3], Lewis and White [4] and Inazumi [5] have proposed the various methods using modified enthalpy, but little attention has been paid to the system of which there exist mass-transfer resistance in the liquid phase. In this paper, the air–aqueous solution system, in which the vapour–liquid equilibria depend upon the temperature and concentration of the liquid, is analysed by a double film model and the relations among the film coefficients and the estimation of interface state are presented on the chart of enthalpy defined as the functions of the temperature and concentration of the liquid.



(a) Counter flow (b) Parallel flow

FIG. 1. Schematic diagram of equipment.

BASIC EQUATIONS OF TRANSFER

Consider the operation of the counter or parallel flow as shown in Fig. 1. The equations which express the mass and the enthalpy balance for the total equipment or a differential height of the equipment can be given as follows:

$$\epsilon \,.\, G\,(H_{G_1} - H_{G_2}) = L\,(C_{L_1} - C_{L_2}), \tag{1}$$

$$\epsilon \,.\, G\,\mathrm{d}H_G = L \,.\, \mathrm{d}C_L, \tag{2}$$

$$\epsilon \,.\, G\,(i_{G_1} - i_{G_2}) = L \,.\, C_s\,(t_{L_1} - t_{L_2}), \tag{3}$$

$$\epsilon \,.\, G\,\mathrm{d}i_G = L \,.\, C_s\,\mathrm{d}t_L. \tag{4}$$

The transfer rate of sensible heat from the bulk of air to the interface

$$\mathrm{d}Q_s = -\,\epsilon \,.\, GC_H\,\mathrm{d}t_G = h_G aS\,(t_G - t_i)\,\mathrm{d}Z. \tag{5}$$

The transfer rate of vapour from the bulk of air to the interface

$$\mathrm{d}W = -\,\epsilon \,.\, G\,\mathrm{d}H_G = k_G aS\,(H_G - H_i)\,\mathrm{d}Z. \tag{6}$$

Enthalpy of the humid air and that at the interface between air and liquid are defined as follows:

$$i_G = C_H \,.\, t_G + \gamma \,.\, H_G \tag{7}$$

and

$$i_i = C_{Hi}\,t_i + \gamma \,.\, H_i. \tag{8}$$

In the gas phase of the air–water system it is assumed that the relation of Lewis is valid, that is

$$\frac{h_G}{k_G} \doteqdot C_H. \tag{9}$$

The transfer rate of enthalpy from the bulk of air to the interface between gas and liquid may be expressed as the following formula:

$$\mathrm{d}Q = \mathrm{d}Q_s + \mathrm{d}Q_L = \mathrm{d}Q_s + \gamma \cdot \mathrm{d}W, \tag{10}$$

i.e.

$$\mathrm{d}Q = -\epsilon \cdot G\,\mathrm{d}i_G \doteqdot k_G a S\,(i_G - i_i)\,\mathrm{d}Z, \tag{11}$$

where it is assumed that C_H is nearly equal to C_{Hi}. Now the rate of enthalpy transfer from the interface to the bulk of liquid is

$$\mathrm{d}Q = -L \cdot C_s\,\mathrm{d}t_L = h_L a S\,(t_i - t_L)\,\mathrm{d}Z \tag{12}$$

and the rate of mass transfer from the interface to the bulk of liquid is

$$\mathrm{d}W = -L \cdot \mathrm{d}C_L = k_L a S\,(C_i - C_L)\,\mathrm{d}Z. \tag{13}$$

The double film model contains two assumptions: these are

(i) Individual film coefficients are constant at all parts through the equipment.

(ii) There is an equilibrium state at the interface between gas and liquid (Fig. 2).

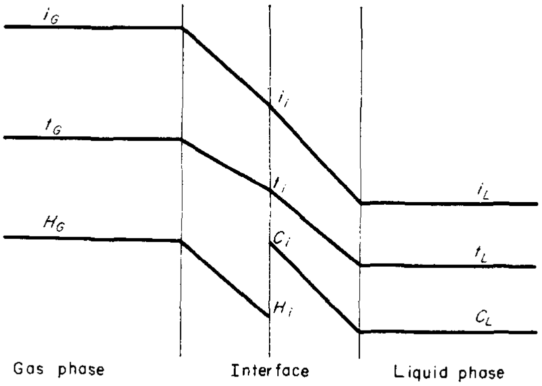


FIG. 2. Concentration and temperature profiles.

$$i_i = f(C_i, t_i), \tag{14}$$

$$H_i = g\,(C_i, t_i). \tag{15}$$

Under these circumstances, well-known equations are defined from equations (5), (6), (11), (12) and (13).

$$N_{Gh} = \epsilon \int_{t_{G2}}^{t_{G1}} \frac{\mathrm{d}t_G}{t_G - t_i}, \tag{16}$$

$$N_{Gm} = \epsilon \int_{H_{G2}}^{H_{G1}} \frac{\mathrm{d}H_G}{H_G - H_i} = \epsilon \int_{i_{G2}}^{i_{G1}} \frac{\mathrm{d}i_G}{i_G - i_i}, \tag{17}$$

$$N_{Lh} = \int_{t_{L2}}^{t_{L1}} \frac{\mathrm{d}t_L}{t_i - t_L}, \tag{18}$$

$$N_{Lm} = \int_{C_{L2}}^{C_{L1}} \frac{\mathrm{d}C_L}{C_i - C_L}, \tag{19}$$

$$H_{Gh} = \frac{G \cdot C_H}{h_G a S} = \frac{Z}{N_{Gh}}, \tag{20}$$

$$H_{Gm} = \frac{G}{k_G a S} = \frac{Z}{N_{Gm}}, \tag{21}$$

$$H_{Lh} = \frac{L C_s}{h_L a S} = \frac{Z}{N_{Lh}}, \tag{22}$$

$$H_{Lm} = \frac{L}{k_L a S} = \frac{Z}{N_{Lm}}, \tag{23}$$

where it is assumed that G, L, C_H, C_s, a and S are constant.

ESTIMATION OF THE INTERFACE STATE AND GAS STATE

From equations (11) and (12) the following relation is given:

$$\frac{i_G - i_i}{t_L - t_i} = -\frac{h_L a}{k_G a}. \tag{24}$$

Similarly, from equations (6) and (13):

$$\frac{H_G - H_i}{C_L - C_i} = -\frac{k_L a}{k_G a}. \tag{25}$$

Applying the relation of Lewis to equations (5) and (11) we have:

$$\frac{\mathrm{d}i_G}{\mathrm{d}t_G} = \frac{i_G - i_i}{t_G - t_i}. \tag{26}$$

From equations (5), (6) and (9):

$$\frac{\mathrm{d}H_G}{\mathrm{d}t_G} = \frac{H_G - H_i}{t_G - t_i}. \tag{27}$$

Since the equilibrium curve may be expressed in the relation of *t–C–H* or *t–C–i*, two variables out of three must be known in order to determine the interface state. For this reason the

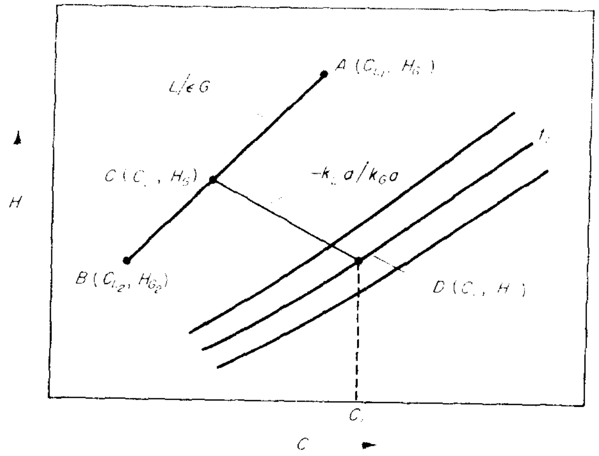


FIG. 3. C–H chart.

two charts expressing the relation of variables which are mutually independent are necessary to determine the interface state. In this case, besides the t–i chart or t–H chart, the C–H chart must be used to determine the interface state between gas and liquid.

Figure 3 shows the C–H chart and the operating line expresses equation (2) and the equilibrium curves correspond to equation (15). The state of gas and liquid at any points through the equipment is the function of the four variables t_G, H_G, t_L and C_L, and as there is a relation (7) among the three variables t_G, H_G and i_G in the gas phase, the other may be determined if two of them are known.

Now in Fig. 3, if the gas temperature t_G corresponding to the arbitrary point C (C_L, H_G) on the operating line is known, the corresponding point $C(t_L, i_G)$ in Fig. 4 can be determined and the inverse process is possible. If a tie line whose slope is $-k_La/k_Ga$ is drawn from the point C in Fig. 3 while a tie line whose slope is $-h_La/k_Ga$ is drawn from the point C in Fig. 4, the point D, which represents the interface state, can be determined on an equilibrium curve as the point where each of t_i, C_i, H_i and i_i has the same value in both figures. For example, when the gas state C', corresponding to the point C, is known in Fig. 4, if C_i is assumed, t_i and i_i (H_i) are obtained from the point D on the equilibrium curve and then C_i can be determined in order that H_i obtained from Fig. 4, coincides with H_i obtained from t_i and C_i in Fig. 3.

As described above, it is necessary that the corresponding gas state C' must be known in order to determine the interface state between gas and liquid at an arbitrary point through the equipment, therefore the determinations of the interface and gas states must be carried out simultaneously. The gas state can be determined by a graphical method as shown in Fig. 4. The interface state D is determined from the points A (t_{L_1}, i_{G_1}) and A' (t_{G_1}, i_{G_1}) and as the line AD has the slope $\mathrm{d}i_G/\mathrm{d}t_G$, C' is obtained as the joining point of the line AD and the line of $i = i_G$ and by succeeding these steps the variations of gas state will be followed up.

RELATIONS BETWEEN OVERALL COEFFICIENT AND INDIVIDUAL FILM COEFFICIENTS

Enthalpy i_L, which is in equilibrium with the bulk concentration C_L and the bulk temperature t_L of liquid, is given from equation (14).

$$i_L = f\,(C_L, t_L). \tag{28}$$

The overall mass-transfer coefficient, $K_{0G}a$, is defined by taking the enthalpy difference between the bulk of gas and liquid as the driving force, as in the following equation:

$$\mathrm{d}Q = -\,\epsilon G\,\mathrm{d}i_G = K_{0G}aS\,(i_G - i_L)\,\mathrm{d}Z. \tag{29}$$

Now, assuming that the equilibrium curve can be represented by a straight line, that is

$$H_i = m' \cdot t_i + n' \cdot C_i + b', \tag{30}$$

$$i_i = m \cdot t_i + n \cdot C_i + b, \tag{31}$$

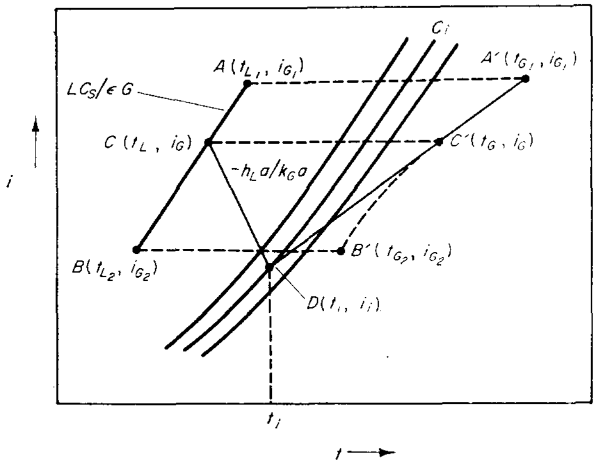


FIG. 4. *t–i* chart.

$$i_L = m \cdot t_L + nC_L + b. \tag{32}$$

From equation (8),

$$m = C_{H_i} + \gamma m', \quad n = \gamma n', \quad b = \gamma \cdot b', \tag{33}$$

where the average value is used for C_{H_i}.

Combining equations (31) and (32),

$$i_G - i_L = (i_G - i_i) + (i_i - i_L) = (i_G - i_i) + \\ + m(t_i - t_L) + n(C_i - C_L). \tag{34}$$

From equations (6) and (13),

$$\mathrm{d}W = k_GaS(H_G - H_i)\,\mathrm{d}Z \\ = k_LaS(C_i - C_L)\,\mathrm{d}Z. \tag{35}$$

From equations (29), (11) and (12)

$$\mathrm{d}Q = K_{0G}aS(i_G - i_L)\,\mathrm{d}Z = k_GaS(i_G - i_i)\,\mathrm{d}Z \\ = h_LaS(t_i - t_L)\,\mathrm{d}Z. \tag{36}$$

Defining that the ratio of overall enthalpy transfer rate to sensible heat-transfer rate in equation (10) is α, from equations (6), (11), (12) and (13),

$$\alpha = \frac{\mathrm{d}Q}{\mathrm{d}Q_L} = \frac{\mathrm{d}Q}{\gamma\,\mathrm{d}W} = \frac{i_G - i_i}{\gamma(H_G - H_i)} \\ = \frac{h_L(t_i - t_L)}{\gamma k_L \cdot (C_i - C_L)} = \frac{h_L(t_i - t_L)}{\gamma \cdot k_G(H_G - H_i)}. \tag{37}$$

Using α, from equations (35) and (37), the following equation is given:

$$\mathrm{d}Q = \gamma \cdot \alpha \cdot \mathrm{d}W = \gamma \cdot \alpha \cdot k_LaS(C_i - C_L)\,\mathrm{d}Z. \tag{38}$$

Also, from the equations (36) and (38);

$$K_{0G}a(i_G - i_L) = k_Ga(i_G - i_i) \\ = h_La(t_i - t_L) = \gamma \cdot \alpha k_La(C_i - C_L)$$

$$\therefore \frac{i_G - i_L}{1/K_{0G}a} = \frac{i_G - i_i}{1/k_Ga} = \frac{t_i - t_L}{1/h_La} = \frac{C_i - C_L}{1/\gamma.\alpha k_La} \\ = \frac{(i_G - i_i) + m(t_i - t_L) + n(C_i - C_L)}{1/k_Ga + m/h_La + n/\gamma\alpha k_La}. \tag{39}$$

Therefore, the relation between the overall coefficients $K_{0G}\,a$ and individual film coefficients k_Ga, h_Ga and k_La can be expressed as the following equation from equations (34) and (39)

$$\frac{1}{K_{0G}a} = \frac{1}{k_Ga} + \frac{m}{h_La} + \frac{n}{\gamma.\alpha.k_La}. \tag{40}$$

Equation (40) is the local relation which is valid at any point Z through the equipment. It has been found that the overall coefficient $K_{0G}a$ is not constant, because the dimensionless ratio α is generally a function of the location even though k_G, h_G, k_L, h_L, m and n are constant. In the next discussion, overall relations between coefficients will be obtained considering this fact. Integrating equation (29)

$$\frac{SZ}{G} = \epsilon \int_{i_{G_2}}^{i_{G_1}} \frac{\mathrm{d}i_G}{K_{0G}a(i_G - i_L)}. \tag{41}$$

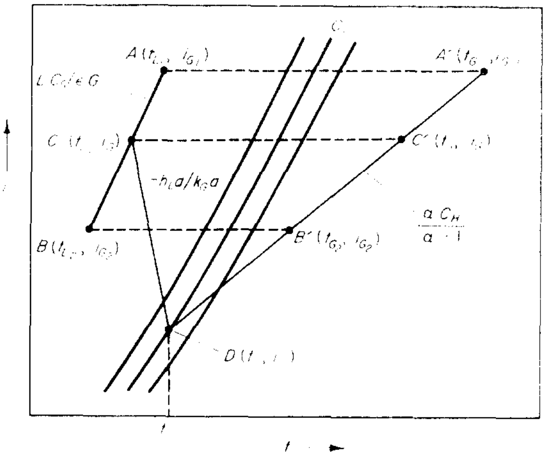


FIG. 5. Representation of the graphical method for the case α = const.

Average value $(K_{0G}a)_m$ is defined as follows:

$$\frac{1}{(K_{0G}a)_m} = \int_{i_{G_2}}^{i_{G_1}} \frac{di_G}{K_{0G}a(i_G - i_L)} \Bigg/ \int_{i_{G_2}}^{i_{G_1}} \frac{di_G}{i_G - i_L}. \tag{42}$$

Using this definition,

$$(N_{0G})_m = \epsilon \int_{i_{G_2}}^{i_{G_1}} \frac{di_G}{i_G - i_L}, \tag{43}$$

$$(H_{0G})_m = \frac{G}{(K_{0G}a)_m \cdot S} = \frac{Z}{(N_{0G})_m}. \tag{44}$$

Taking the average value of both sides of equation (40),

$$\frac{1}{(\alpha)_m} = \int_{i_{G_2}}^{i_{G_1}} \frac{di_G}{\alpha\,(i_G - i_L)} \Bigg/ \int_{i_{G_2}}^{i_{G_1}} \frac{di_G}{i_G - i_L} \tag{45}$$

can be defined. From these definitions the following overall relation can be given:

$$\frac{1}{(K_{0G}a)_m} = \frac{1}{k_G a} + \frac{m}{h_L a} + \frac{n}{\gamma\,(\alpha)_m\,k_L a}. \tag{46}$$

If α is constant at any point through the equipment, from equations (6), (11) and (37),

$$\alpha = \frac{di_G}{\gamma \cdot dH_G} = \frac{i_{G_1} - i_{G_2}}{\gamma\,(H_{G_1} - H_{G_2})}. \tag{47}$$

Therefore, only in this case $K_{0G}a = (K_{0G}a)_m$, $\alpha = (\alpha)_m$, so $K_{0G}a$ is constant through the equipment and the equation (30) coincides with equation (46).

From equation (10)

$$\alpha = \frac{dQ}{dQ_L} = \frac{dQ_s + dQ_L}{dQ_L} = \frac{dQ_s}{dQ_L} + 1.$$

Substituting equations (5) and (6) into the above equation and using equation (9)

$$\frac{H_G - H_i}{t_G - t_i} = \frac{C_H}{\alpha - 1} \cdot \frac{1}{\gamma}. \tag{48}$$

From equations (26), (37) and (48)

$$\frac{di_G}{dt_G} = \frac{i_G - i_i}{t_G - t_i} = \frac{\alpha C_H}{\alpha - 1}. \tag{49}$$

If α is constant,

$$\frac{di_G}{dt_G} = \frac{i_{G_1} - i_{G_2}}{t_{G_1} - t_{G_2}} = \text{const.} \tag{50}$$

Therefore the variations of the gas state are represented by a straight line and as α is obtained from equation (47) the value of equations (49) and (50) is determined and then the interface state is represented as the joining point of the line of equations (24) and (49). In this case trial and error methods are not necessary. Now, in the air–water system, as $n = 0$ and $n' = 0$ in equations (30) and (31) and $k_L a \to \infty$, then

$$\frac{1}{K_{0G}a} = \frac{1}{k_G a} + \frac{m}{h_L a}. \tag{51}$$

Therefore, it follows that α has the significant meaning as the value expressing the transfer characteristics of the system in which there exists the mass-transfer resistance in the liquid phase and in which the interface state is a function of temperature and concentration.

SUMMARY AND CONCLUSIONS

The interface state and the gas state of the system in which the mass-transfer resistance exists in the liquid phase can be determined by a graphical method using the *C–H* chart, the *t–i* chart, and the relations between film coefficients as shown in Figs. 3 and 4.

Provided that the vapour–liquid equilibria can be represented by linear relations, the relations between the overall coefficient and individual film coefficients can be expressed by equation (40).

Since α, which indicates the ratio of total heat-transfer rate to sensible heat-transfer rate, is generally a function of k_G, h_L, k_L, m and n, so $K_{OG}a$ as defined by equation (29) is not always constant through the equipment. It has been found that if α is constant $K_{OG}a$ may be constant through the equipment.

The water–air system in which the mass-transfer resistance in liquid phase is absent can be recognized as a special case of a gas–solution system.

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Résumé—La méthode d'analyse par le modèle du double film est étendue au cas général du système air-solution aqueuse, et on obitent les relations entre les coefficients de transport du film gazeux et liquide. De plus, on discute l'application de l'opération de déhumidification à la conception de l'équipement.

Zusammenfassung—Die Analysis nach dem Doppelfilmmodell wird auf den allgemeinen Fall von Luft-Wassergemischen erweitert und Beziehungen zwischen den Übergangskoeffizienten für Gas und Flüssigkeitsfilm abgeleitet. Daneben wird die Anwendung des Entfeuchtungsvorganges für den Entwurf von Apparaten besprochen.

Аннотация—Метод анализа двойной пленочной модели применяется к общему случаю системы вода-воздух. Выведены соотношения между коэффициентами переноса в газовой и жидкой пленке. Кроме того, рассматривается применение этого метода при конструировании оборудования для обезвоживания.